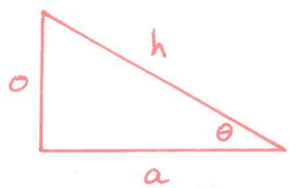
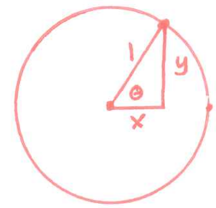
Sec. 7.4 The Tangent Function

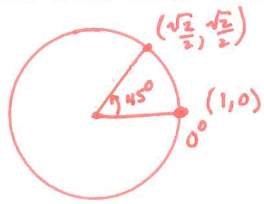
In a right triangle: $\tan\theta = \dfrac{opposite}{adjacent}$



On a unit circle: $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{y}{x}$



Ex. Find the tan of 0°, 45°, and 150° (use unit circle).



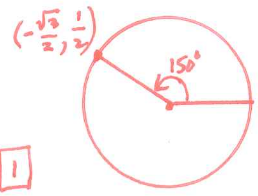
$\tan 0° = \frac{y}{x}$

$= \frac{0}{1}$

$\tan 0° = \boxed{0}$

$\tan 45° = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$\tan 45° = \boxed{1}$



$\tan 150° = \dfrac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$= \frac{1}{2} \cdot -\frac{2}{\sqrt{3}}$

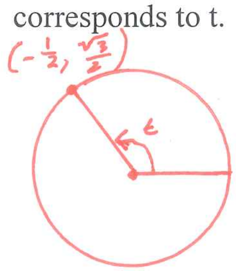
$\tan 150° = \boxed{-\frac{1}{\sqrt{3}}}$

Ex. Find the exact value of each expression without a calculator: $\tan 20° - \dfrac{\sin 20°}{\cos 20°}$

$\tan 20° - \tan 20°$

0

Ex. Let t be a real number and P = $(\dfrac{-1}{2}, \dfrac{\sqrt{3}}{2})$ be the point on the unit circle that corresponds to t. Find the trigonometric values for sin, cos and tan.



$\cos t = \boxed{-\frac{1}{2}}$

$\sin t = \boxed{\frac{\sqrt{3}}{2}}$

$\tan t = \dfrac{\sin\theta}{\cos\theta}$

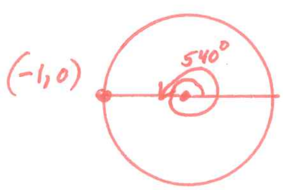
$= \dfrac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$\frac{\sqrt{3}}{2} \cdot -\frac{2}{1}$

$= \boxed{-\sqrt{3}}$

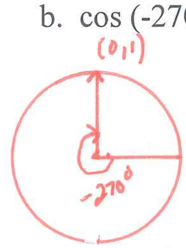
Ex. Find the exact value of the following:

a. sin (540°) $360° + 180°$



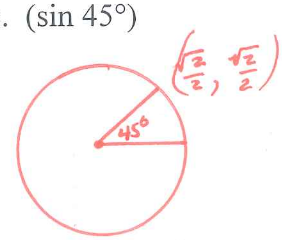
$\sin 540° = \boxed{0}$

b. cos (-270°)



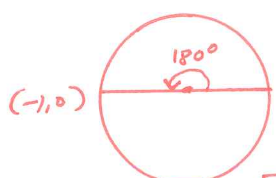
$\cos(-270) = \boxed{0}$

c. (sin 45°)



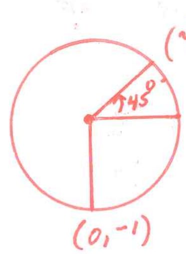
$\sin 45° = \boxed{\frac{\sqrt{2}}{2}}$

d. (cos 180°)



$\cos 180° = \boxed{-1}$

e. tan 45° – sin 270°



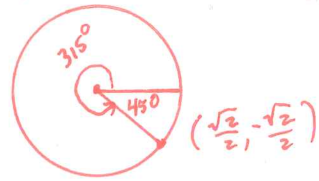
$\tan 45 = 1$

$\sin 270° = -1$

$1 - -1$

$\boxed{2}$

f. tan 315°



$\tan 315° = \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$\tan 315° = \boxed{1}$

Periodic Properties:

$\sin(\theta + 2\pi k) = \sin\theta$	$\cos(\theta + 2\pi k) = \cos\theta$	$\tan(\theta + \pi k) = \tan\theta$
$\csc(\theta + 2\pi k) = \csc\theta$	$\sec(\theta + 2\pi k) = \sec\theta$	$\cot(\theta + \pi k) = \cot\theta$

Ex. Find the exact values using periodic properties of:

a. $\sin 765^\circ$

$\sin(45^\circ + 720^\circ)$

$\sin 45^\circ = \boxed{\frac{\sqrt{2}}{2}}$

b. $\tan 225^\circ$

$\tan 225^\circ$

$\tan(45^\circ + 180^\circ)$

$\tan 45^\circ = \boxed{1}$

c. $\cos 900^\circ$

$\cos(180^\circ + 720^\circ)$

$\cos 180^\circ = \boxed{-1}$

Properties of the Tangent Function (Derive cotangent from this):

1. The domain is the set of all real numbers except odd multiples of 90°.
2. The range is the set of all real numbers.
3. The tangent function is an odd function with symmetry with respect to the origin.
4. The tangent function is periodic with period 180°.
5. The x-intercepts are …-360°, -180°, 0, 180°, 360°, …
6. The y-intercept is 0.
7. Vertical asymptotes occur at $x = -270^\circ, -90^\circ, 90^\circ, 270^\circ$...

$\tan 0^\circ = \frac{0}{1} = 0$

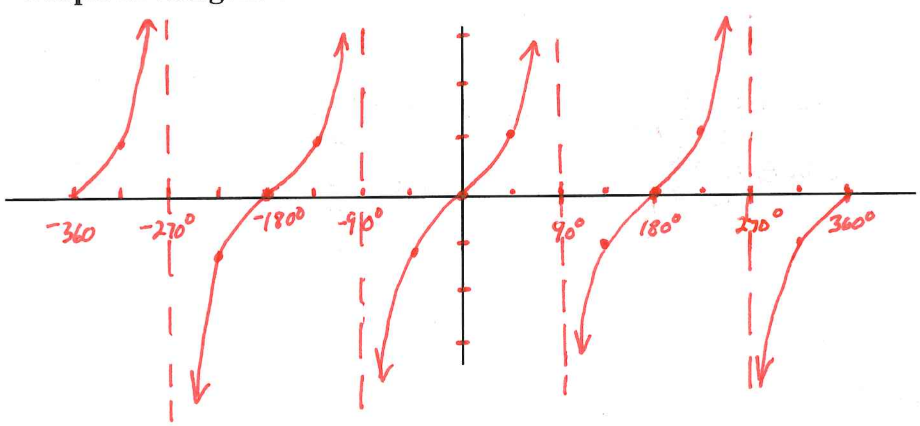
$\tan 45^\circ = 1$

$\tan 90^\circ = \frac{1}{0} =$ undefined

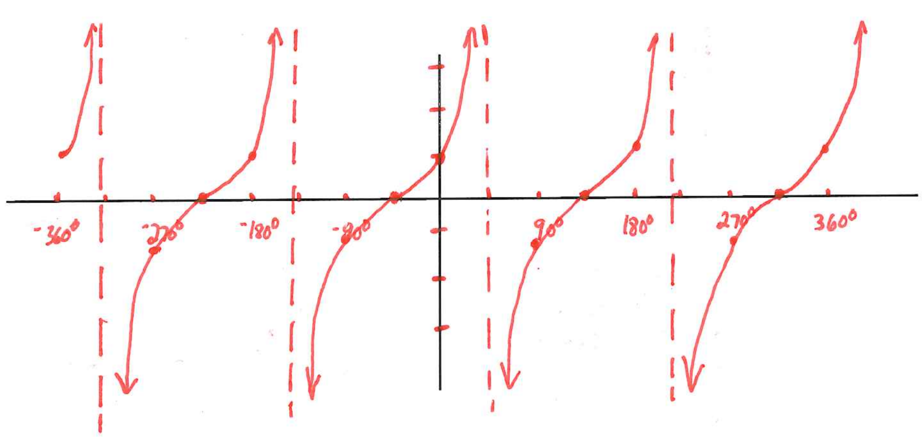
$\tan 135^\circ = -1$

$\tan 180^\circ = \frac{0}{-1} = 0$

Graph of Tangent of x:

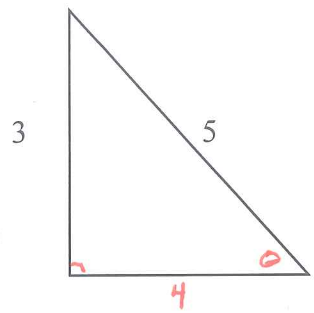


Ex. Graph $y = -\tan(x + 45^\circ)$ by hand using the graph of the tan x.



NOTE: **tan θ = $\frac{\text{opposite}}{\text{adjacent}}$**

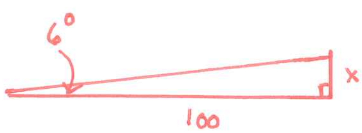
Ex. Find the exact value of the three trigonometric functions if:



$\sin\theta = \frac{o}{h}$ $\cos\theta = \frac{a}{h}$ $\tan\theta = \frac{o}{a}$

$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

Ex. The grade of a road is calculated from its vertical rise per 100 feet. For instance, a road that rises 8 ft in every one hundred feet has a grade of 8% (8/100). Suppose a road climbs at an angle of 6 degrees to the horizontal. What is its grade?



$\tan 6^\circ = \frac{x}{100}$

$x = 100 \tan 6^\circ$

$\boxed{x = 10.5104\%}$

Interpreting the Tangent Function as Slope:

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y-0}{x-0} = \frac{y}{x} \text{ so } \text{Slope} = \tan\theta.$$

(If and only if the line passes through the origin.)

HW: pg 295-296, #1-5,7,11-25,27,29,32